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Entrainment by a plume or jet at a density interface

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The rate of entrainment through the end of a plume or jet which impinges on a density interface has been determined in the laboratory, where the interface was produced by layers of fresh and salt water and the plume by a salt-water source. Observations of the impingement area indicated that entrainment was confined to a region about the size of the plume cross-section. It was thus concluded that the entrainment flux into the plume must be a function of the local width, velocity and buoyancy difference, and these can be combined into a single parameter, the Froude number. Measurements of the volume flux showed it to be proportional to the cube of the Froude number. The flux of buoyancy from the salt- to fresh-water volumes is consequently proportional to the Froude number. In the second part of the study the density distribution in the initially fresh layer was derived and is verified by the experiments. This distribution has direct applications in the analysis of convective motions in the atmosphere and the ocean.

1. Introduction

There are many instances in nature where a plume or jet impinges on a density interface and does not penetrate it. The result is a termination of the forward motion of the turbulent fluid and a steady lateral spreading of the plume or jet fluid along the interface. One example is the plume from an industrial chimney rising underneath an atmospheric inversion on a very calm day. The pollutant in the plume provides a marker for the buoyant fluid and shows that the plume effectively ends at the inversion with a curved cap. The pollutant stays below the inversion and slowly spreads radially along it. The plume motion up to the inversion can be analysed in terms of the bulk properties of the source, that is, the mass and buoyancy flux, by using equations of conservation and the entrainment assumption of G. I. Taylor. The radial spreading can be similarly analysed. However, the volume flux of lighter fluid entrained by the plume in the impingement area has not been determined. The object of this experimental study is to measure it in terms of the properties of the plume or jet.

A source of momentum M_0 or of buoyancy F_0 is supposed to be introduced in a fluid layer of thickness H and density ρ_{a0} which is above a second fluid layer of density ρ_2 of very great depth. If turbulent stresses are assumed to dominate the flow in the plume, then the volume flux Q^* entrained from the second layer by the plume is a function only of the three variables $\rho_2 - \rho_{a0}$, F_0 and H, which can be combined in a single parameter. Complications to this picture are introduced by the experimental technique. The density interface was produced in a glass-sided tank by placing a layer of fresh water over a layer of salt water. A small pipe produced a jet of fresh water for the jet experiments and a jet of salt water for the plume experiments. Dye was introduced into the jet to observe the entrainment process. With this technique it proved possible to measure Q^* and the density distribution with relative ease. The problem arose because the density of the initially fresh-water layer increased steadily as more salt water was introduced by the source and by the entrainment at the interface. The density distribution ρ_a across the fresh-water layer is therefore a function of both position and time. An analytical description of this distribution is contained in §4 of the paper along with experimental confirmation.

2. Entrainment at the interface

On leaving the source the plume expanded linearly like that from a point source. This is evident on figure 1 (*a*) (plate 1), which shows the clearly defined but irregular edge. The outside width measured on the photograph was found to be $2{\cdot}8b$, where b is the nominal plume radius used by Baines & Turner (1969). For its definition the velocity and density profiles are fitted with a Gaussian profile and b is the radius where the property has a value 0·368 of the maximum. As the plume impinged on the interface it was observed to entrain fluid through its end over a relatively small area. This entrained fluid mixed with the plume fluid within a small volume, i.e. about the volume of a large eddy. Upon impact with the interface the large rotating lumps of fluid, i.e. the largest eddies, stopped their forward motion and moved out along the interface. These can be seen clearly in figure 1 (*a*) but in figure 1 (*b*) are not evident. The rotational motion ceased before the eddy had moved laterally a distance of $2b$ and the spreading from this time onward was a laminar motion. A few striations in the dyed layer were the only evidence of the plume turbulence.

When dye was not present in the plume a circular depression with a rough surface was observed in the heavy fluid immediately underneath the plume, that is, the stagnation pressure was balanced by a buoyant force. It was also observed that within an area of radius about $2b$ wisps of the heavier fluid were moving upwards into the spreading plume fluid. These appeared randomly over the surface and disappeared before a height b above the interface was reached. These observations suggested that the mechanism of entrainment is primarily by large eddies scooping heavy fluid between them as each intersects another roughly spherical eddy. An alternative explanation would be that viscous stresses between the eddies and the heavier fluid accelerate this heavier fluid and carry it up into the turbulent region. If this had been the case, viscosity would be a variable to be considered in the analysis. The experimental results indicated that the Reynolds number of the plume had no effect on the entrainment flux. It is thus concluded that the alternative explanation is not valid.

It is concluded from these observations that the entrainment depends only on the characteristics of the plume or jet as it impinges and on the density difference across the interface. The set of local characteristics of the plume consists of its centre-line velocity w_1, radius b_1 and buoyancy Δ_1 but for a plume from a point source only two are independent; w_1 and b_1 were chosen. The density difference across the interface is the other characteristic, and is naturally described by the buoyancy $\Delta_2 = g(\rho_2 - \rho_1)/\rho_{a0}$. It was found that the smaller this density difference was the deeper was the depression at the end of the plume and the more intense appeared to be the entrainment process. The three parameters can be combined into one dimensionless parameter, the Froude number $Fr = w_1/(\Delta_2 b_1)^{\frac{1}{2}}$, which can also be interpreted as the square root of the ratio between the momentum of the plume and buoyant force required to lift the fluid from the interface.

The entrainment flux should be defined relative to the same plume characteristics, i.e. as a ratio $Q^*/w_1 b_1$. If the conjecture that the local characteristics determine the entrainment is correct this is a function only of the Froude number for both plumes and jets.

3. Measurement of entrainment flux

All the experiments were performed in a glass-walled tank 30 cm deep, 30 cm wide and 60 cm long. The source fluid was introduced through a glass tube of inside diameter 5 mm in which a piece of wire mesh had been placed to promote transition from laminar to turbulent flow. It was found in every case that the virtual source was coincident with the end of the tube. The method used to locate the virtual source was that described by Baines & Turner (1969). At the start of the experiment the source was placed at the free surface in the centre of the tank. The salt water for the plume was a mixture which had been previously adjusted to a specified density and allowed to come to room temperature, i.e. the temperature of the tank fluid. The interface was produced by initially filling the tank with a thick layer of fresh water and then slowly introducing a salt-water layer beneath it. The fresh water had been allowed to stand for several hours after filling to dissipate the vorticity. The salt layer was usually dyed deep blue.

A volume flowmeter in the line leading to the source enabled the discharge flux Q_0 to be determined. The kinematic momentum flux M_0 from the source was found by multiplying Q_0 by the average velocity, and the buoyancy flux F_0 was found by multiplying Q_0 by Δ_0, the buoyancy of the source fluid. There is a possibility of error in this method if the velocity across the outlet is not constant. This could not be measured but a nozzle was used as the outlet to ensure that this condition was approached. The volume flux Q^* was determined by measuring the rate of change of the elevation of the interface. A pair of parallel scales was used to eliminate parallax. The elevation of the interface could always be accurately determined because of the difference between the colours and refractive indices of the two fluids. As noted above, the dimensionless representation of Q^* required its division by $w_1 b_1$. These parameters were calculated using the equation for a point source in a stratified medium as presented by Baines & Turner (1969) but revised following the analysis in §4. It was found that these values and those calculated using the equations for a point source in a uniform environment always differed by very small amounts. It was, however, necessary to specify the entrainment velocity relative to w, the centre-line velocity. For plumes, this was determined directly in the experiment. A small amount of dye was introduced from the source at the start of the experiment, and this formed a coloured front which moved upwards through the fresh-water layer. The velocity of this front could be used to define α as shown by Baines & Turner (1969). In the thirty separate experiments it was found that α varied from 0·084 to 0·100 with no discernible pattern. The mean value of 0·093 is the same as that found by Morton, Taylor & Turner (1956) but it is appreciably less than the value of 0·100 found by Baines & Turner in an experimental apparatus of about the same size. This value of 0·093 was used for all analyses because it represents the average. For the case in which the source fluid was fresh water, the reference velocity and radius were assumed to be those produced by a source of momentum in the uniform fluid with the entrainment constant $\alpha = 0{\cdot}057$, the value found by Albertson *et al.* (1950).

The dimensionless entrainment flux is presented on figure 2 as a function of the Froude number. Symbols identify the values of the source discharge flux and buoyancy $\Delta_0 = g(\rho_0 - \rho_{a0})/\rho_{a0}$ and the depth of the fresh-water layer for each experiment. These and the density of the salt-water layer were varied over as large a range as possible. The results for both the plume and jet appear to follow the same curve, indicating that Q^* is indeed defined by the local characteristics of the turbulent flow. Following this deduction, it can be said that the entrainment is not affected by the motion within either the lighter or heavier fluids but is determined only by the characteristics of the turbulence within the plume or jet at impingement. Another important property of the flow is the depth of penetration into the heavy fluid layer. This was barely perceptible at small Fr but for $Fr > 2$ the plume or jet penetrated deeply. The eddies within the plume were in contact with the heavier fluid for a much longer time, which may be significant in explaining the variation of the entrainment flux with Fr and why it differs from the relationship presented by Kato & Phillips (1969). They showed that if the mean energy of the turbulent flow is the potential energy required to lift the heavier fluid by the distance b_1 a proportionality between the entrainment flux and the square of the Froude number results. The results on figure 2 show a proportionality to the cube. This is exactly the same form as that found by Turner (1968), who used mechanical agitation within a fresh-water layer to produce turbulence. He measured the velocity of entrainment directly from the increase in the salt concentration within the initially fresh water. Results were presented in non-dimensional form but with arbitrary length and velocity scales. Thompson (1969, private communication) has measured the intensity u' and scale L of turbulence in the tank used by Turner and so has been able to assign exact values to the parameters w^*/w_1 and $u'/(L\Delta_2)^{\frac{1}{2}}$. An attempt has been made using Thompson's results to compare Turner's results with figure 2. This required the definition of the intensity and scale of turbulence for the plume or jet and the derivation of the entrainment velocity w^* from the entrainment flux Q^*. No measurements of turbulence within plumes are available but the free jet is well documented. The results of Wygnanski & Fiedler (1969) show a unique variation for the range of jet development used in this experiment. However, turbulence in the jet is not isotropic so a choice must be made of the component of the

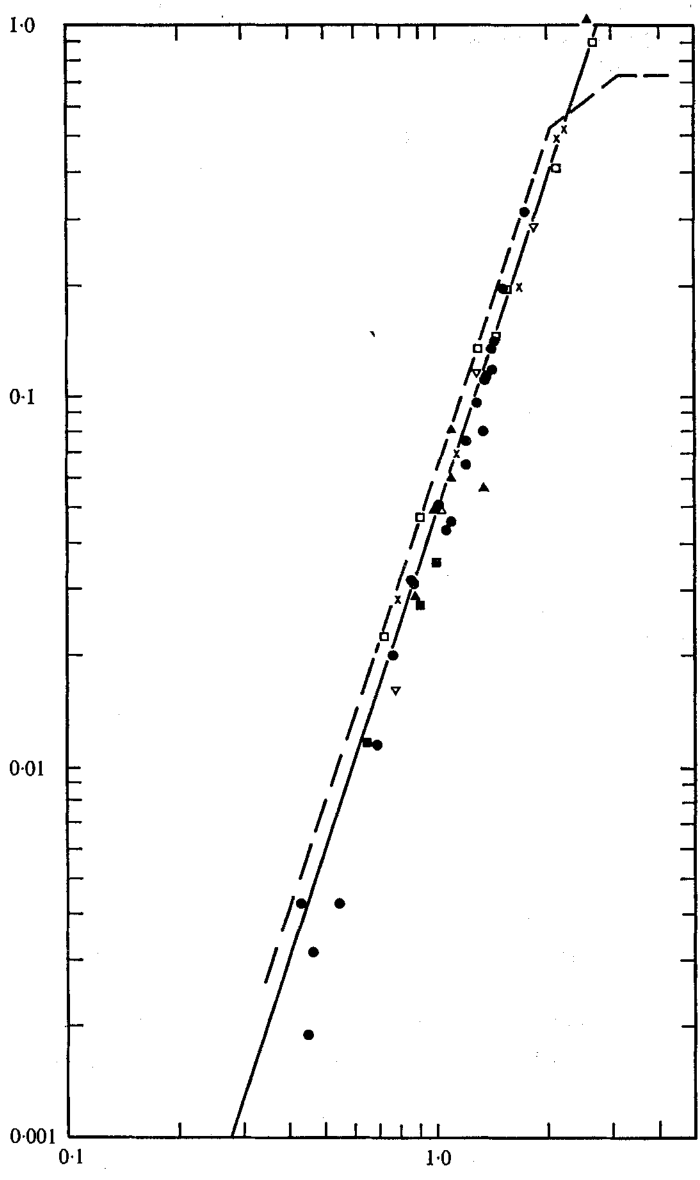

FIGURE 2. Entrainment volume flux for plumes and jets. - - -, mean of data of Turner (1968) transformed on basis of turbulent velocity and scale; ——, best-fit line with slope of 3.

	●	■	▲	□	△	▽	×
Q_0	1	0·25	2·3–4·5	1	0·31	1	Jet
H (cm)	22	22	22	16	15	11–22	
Δ_0	120	120	120	120	120	10	

intensity and the particular integral scale to be used. In determining the dashed line shown on figure 2 the mean of the three components of the intensity and the mean of the three integral scales were expressed as ratios to the maximum velocity w_1 and the radius b_1, respectively, from the Wygnanski & Fiedler data. Using these, the mean line through Turner's results was converted to w^*/w_1. This entrainment velocity was next converted to an entrainment flux by assuming that the entrainment at the end of the plume occurs over a circle of radius $2b$. The agreement between the line and the measured data on figure 2 must be considered fortuitous because both the area of entrainment and turbulence properties are known only to within an order of magnitude. Nevertheless, it is evident that there is a universal form for the entrainment velocity in terms of the characteristics of either the turbulence or mean flow in the plume or jet.

This variation of the entrainment flux with the cube of the Froude number is also evident in the data reported by Kato & Phillips (1969). The authors claim that the variation follows the square of the Froude number but careful examination of the data, particularly at small Froude numbers, shows that the results are closer to the cube. Both Turner (1968) and Kato & Phillips (1969) found that the entrainment rate varies at a rate less than that of the cube for large Froude numbers but this effect is not evident in data on figure 2.

The Froude number of a plume impinging on an interface reaches a limiting value if the density of the plume as it strikes the interface is exactly equal to that of the heavy fluid. The momentum of the plume carries it through the interface, after which the plume fluid is surrounded by fluid of the same density, the flow hence being that of a jet. For this limiting condition the Froude number is defined in terms of w_1, b_1 and $\Delta_2 = \Delta_1$ and reduces to

$$Fr_{\max} = (5/4\alpha)^{\frac{1}{2}}, \tag{3.1}$$

which can be seen to depend only on α. This relation is easily verifiable by experiment because for Froude numbers less than $Fr_{\max}$ the interface recedes from the source as time progresses and for Froude numbers greater than $Fr_{\max}$ the interface approaches the source. It was found from experiment that the limiting value was 3·80, compared with 3·66 from (3.1). The agreement of the measured and theoretical value is satisfactory considering the uncertainty in the value of α.

Buoyancy flux across the interface

The buoyancy flux F^* is found to be the most important parameter in the determination of the density distribution in the fresh-water layer, as will be noted in §4. By definition

$$F^* = \Delta_2 Q^*, \tag{3.2}$$

and so is determined from the interface motion. It can also be determined from the rate of increase of density at a point in the fresh-water layer. This rate is equal to $F_0 + F^*$ divided by the volume of fresh water. Good agreement between the two methods was found in all of the experiments.

Figure 3 is a plot of F^*/F_0 as a function of Fr for the experiments in which buoyant source fluid was used. The points are scattered about a straight line derived from the best-fit line on figure 2. If this is taken as

$$Q^*/w_1^2 b_1^2 = \text{constant} \times [w_1/(b_1\Delta_2)^{\frac{1}{2}}]^3 \tag{3.3}$$

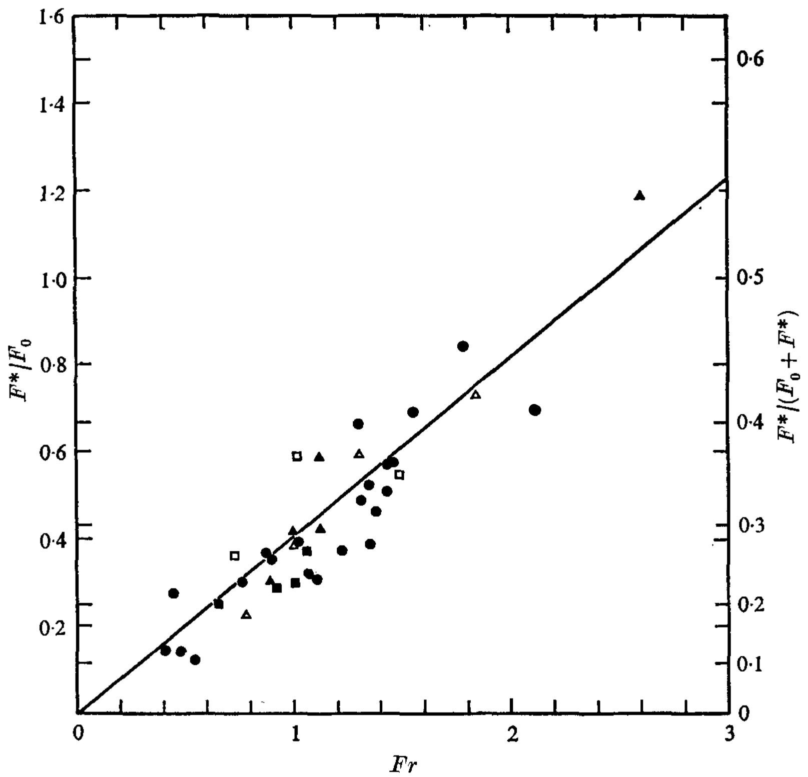

FIGURE 3. Entrainment buoyancy flux for plumes. Symbols as in figure 2.

and (3.3) is multiplied by Δ_2/F_0 then

$$\frac{F^*}{F_0} = \text{constant} \times \left[\frac{w_1^3 b_1}{F_0}\right] \frac{w_1}{(b_1\Delta_2)^{\frac{1}{2}}}, \tag{3.4}$$

which is a linear relationship because the term in square brackets is the square of the Froude number of the plume multiplied by a constant. The analysis of Morton *et al.* (1956) shows that this is constant for all plumes in the region distant from the source. In this set of experiments the plume strikes the interface in this region so the representations on figures 2 and 3 are identical. Such would not be the case if the interface were close to the source.

The vertical scale on figure 3 is presented in two forms. On the left-hand side F^*/F_0, the ratio derived above by considering the entrainment process, is shown. On the right the corresponding values of $F^*/(F^*+F_0) = B$ are given. This ratio of the entrained flux to the total flux entering the environment is significant in the determination of the density distribution in the environment, as shown in the following section.

The buoyancy flux entrained by a jet must be compared with $M_0^{\frac{3}{2}}/H^2$ because F_0 is obviously zero in this case. In terms of this parameter the best-fit line on figure 2 is

$$\frac{F^* H^2}{M_0^{\frac{3}{2}}} = \text{constant} \times \left[\frac{w^3 H^2 b_1}{M_0^{\frac{3}{2}}}\right] \left(\frac{w_1}{(b_1\Delta_2)^{\frac{1}{2}}}\right), \tag{3.5}$$

which is also a linear relationship because the term in square brackets is constant for a jet in the region distant from the source. This was verified by plotting the data from figure 2 for jets using these parameters. The result was agreement similar to that in figure 3.

4. Density distribution in the confined region

If the fresh-water layer is of limited horizontal extent, as it must be in the laboratory, a recirculation flow pattern is developed within it. The plume fluid, after impinging, spreads laterally along the interface until it reaches the edges of the tank. It then moves vertically upwards towards the source. The plume entrains fluid from all levels of this region and induces a pattern of closed streamlines and a density distribution which is not linear and which changes steadily with time. The solution for this density distribution can be obtained by extending the analysis of Baines & Turner (1969), which was for a confined region with impermeable walls.

Basic equations

It is assumed that the density and velocity profiles across the plume are similar at all elevations z below the virtual source and these profiles are of Gaussian form. The equations of conservation of volume, momentum and density deficiency integrated over the plane z = constant reduce to the following set derived by Morton *et al.* (1956) provided that the velocities in the plume are much larger then those in the environment:

$$d(b^2w)/dz = 2\alpha bw, \tag{4.1a}$$

$$d(\tfrac{1}{2}b^2w^2)/dz = b^2\Delta, \tag{4.1b}$$

$$d(\tfrac{1}{2}b^2w\Delta)/dz = b^2w\,\partial\Delta_a/\partial z. \tag{4.1c}$$

Here α is the entrainment constant, chosen to make the rate of entrainment of volume at any height equal to $2\pi b\alpha w$. The density gradient in the environment influences only the last equation. It is defined by

$$\frac{\partial\Delta_a}{\partial z} = \frac{g}{\rho_{a0}}\frac{\partial\rho_a}{\partial z}, \quad \Delta_a = \frac{g(\rho_a-\rho_{a0})}{\rho_{a0}}$$

and is determined by the conservation equation for a scalar property,

$$\partial\Delta_a/\partial t = -w\,\partial\Delta_a/\partial z, \tag{4.2}$$

and the conservation equation for the mass flux of the transporting fluid,

$$-\pi R^2 w = \pi b^2 w, \tag{4.3}$$

where w is the vertical velocity in the environment and it is assumed that $R^2 \gg b^2$, where πR^2 is the cross-sectional area of the tank.

Boundary conditions

The boundary conditions to be imposed depend on the problem to be solved and for one of these entrained fluid is not a factor. The motion of the first front analysed by Baines & Turner (1969) depends only on the entrainment of homogeneous fluid by a plume. In effect, the staining of the first fluid elements released by the source produces a rising front which is independent of any mixing process existing beyond the front. It thus provided the scheme whereby the entrainment constant α discussed in § 3 was determined.

Asymptotic solution

As the experiment progresses the density distribution approaches an asymptotic condition in which density is changing at the same rate at every point. It has been shown that this condition is approached rather quickly. For this asymptotic case the balance of buoyancy flux gives

$$\pi R^2 H \, \partial\Delta_a/\partial t = F_0 + F^* = F_1, \tag{4.4}$$

which defines F_1, the total influx to the confined region. Inserting this equation in (4.2) produces a relation which upon inclusion in (4.1 *c*) allows it to be integrated directly, yielding

$$wb^2\Delta = \frac{2}{\pi} F_1 \left(1 - \frac{z}{H} - \frac{2}{\pi} F^*\right), \tag{4.5}$$

in which the boundary condition on the plume at the virtual source,

$$wb^2\Delta = (2/\pi) F_0 \quad \text{at} \quad z = 0,$$

has been used to evaluate the constant of integration.

At this point it is convenient to convert to dimensionless variables. If the following substitutions are introduced the differential equations are reduced to the simplest form:

$$\Delta = \tfrac{1}{2}\pi^{-\frac{2}{3}} F_1^{\frac{2}{3}} \alpha^{-\frac{4}{3}} H^{-\frac{5}{3}} f(\delta), \tag{4.6a}$$

$$\Delta_a = \tfrac{1}{4}\pi^{-\frac{2}{3}} F_1^{\frac{2}{3}} \alpha^{-\frac{4}{3}} H^{-\frac{5}{3}} (f_0(\delta) - \tau), \tag{4.6b}$$

$$b = 2\alpha H h(\delta), \tag{4.6c}$$

$$u = \pi^{-\frac{1}{3}} F_1^{\frac{1}{3}} \alpha^{-\frac{2}{3}} H^{-\frac{1}{3}} g(\delta), \tag{4.6d}$$

$$\tau = 4\pi^{-\frac{1}{3}} \alpha^{\frac{4}{3}} (H/R)^2 F_1^{\frac{1}{3}} H^{-\frac{4}{3}} t, \tag{4.6e}$$

$$\delta = 2/H. \tag{4.6f}$$

That is, (4.1) become

$$d(gh^2)/d\delta = gh, \tag{4.7}$$

$$d(g^2h^2)/d\delta = h^2 f, \tag{4.8}$$

$$fgh^2 = (1 - \delta) - B. \tag{4.9}$$

A further simplification is possible if the variables are changed to

$$j = -gh^2, \quad k = gh \tag{4.10), (4.11}$$

and (4.7) and (4.8) combined to give

$$\frac{dj}{d\delta} = -k, \quad \frac{dk^2}{d\delta} = \frac{j}{k^2}[B - (1 - \delta)], \tag{4.12), (4.13}$$

whose solutions must satisfy the boundary conditions for a point source, namely $j = 0$ and $k = 0$ at $\delta = 0$. Series solutions of (4.12) and (4.13) can be obtained which converge rapidly over the whole range of integration $0 \leqslant \delta \leqslant 1$. The first five terms of the solutions for j and k are given by

$$j = -0\cdot4598\delta^{\frac{5}{3}}(1-B)^{\frac{1}{3}}\left[1-0\cdot1282\frac{\delta}{1-B}-0\cdot02178\frac{\delta^2}{(1-B)^2}+0\cdot00156\frac{\delta^3}{(1-B)^3}\cdots\right], \tag{4.14}$$

$$k = 0\cdot7663\delta^{\frac{2}{3}}(1-B)^{\frac{1}{3}}\left[1-0\cdot2051\frac{\delta}{1-B}-0\cdot04792\frac{\delta^2}{(1-B)^2}+0\cdot00436\frac{\delta^3}{(1-B)^3}\cdots\right]. \tag{4.15}$$

All the properties of the plume can be determined algebraically from these solutions but finding the density distribution requires the integration of (4.1*c*). In dimensionless form this distribution is

$$f_0 = C+\frac{3\cdot2624}{\delta^{\frac{2}{3}}(1-B)^{\frac{1}{3}}}\left[1-0\cdot2564\frac{\delta}{(1-B)}-0\cdot0193\frac{\delta^2}{(1-B)^2}\right.$$
$$\left.-0\cdot00179\frac{\delta^3}{(1-B)^3}-0\cdot00030\frac{\delta^4}{(1-B)^4}\cdots\right]. \tag{4.16}$$

The constant of integration C can be determined by setting the value of Δ_a in (4.6*b*) at the interface $\delta = 1$ for $t = 0$. The density difference produced by the initial front is very small and can be set to zero without loss of accuracy.

Experimental confirmation

The density distribution in most of the experiments was measured by sampling at points in the initially fresh-water layer. Twenty-five ml of liquid were withdrawn and weighed on a sensitive balance. The density was expressed as a dimensionless ratio consistent with (4.6) after an intermediate calculation of the buoyancy Δ_a. Results for one group of experiments are plotted on figure 4. The co-ordinates have been arranged to correspond to the experimental configuration: the vertical co-ordinate is the dimensionless depth below the source and the horizontal co-ordinate the difference in buoyancy from that just above the interface. The solid line gives the solution for $B = 0\cdot3$ from (4.16) and the plotted points are the measured values for a range of B near $0\cdot3$. There is scatter of the points about the theoretical solution but this is not sufficient to throw doubt on the good agreement. For comparison the solution for $B = 0$, an impermeable lower surface, is plotted as a dashed line on the same figure. The difference between the two lines is much larger than the scatter of the points. This demonstrates that the presence of the interface produces a difference which should be considered in any physical problem. F^* produces steeper density gradients.

5. Discussion

The entrainment through the end of the plume introduces more fluid into the plume and thus would dilute a pollutant within it. This is a further dilution to that produced by the lateral entrainment and its importance in a physical problem depends on the relative size of the two fluxes. Consider two examples from

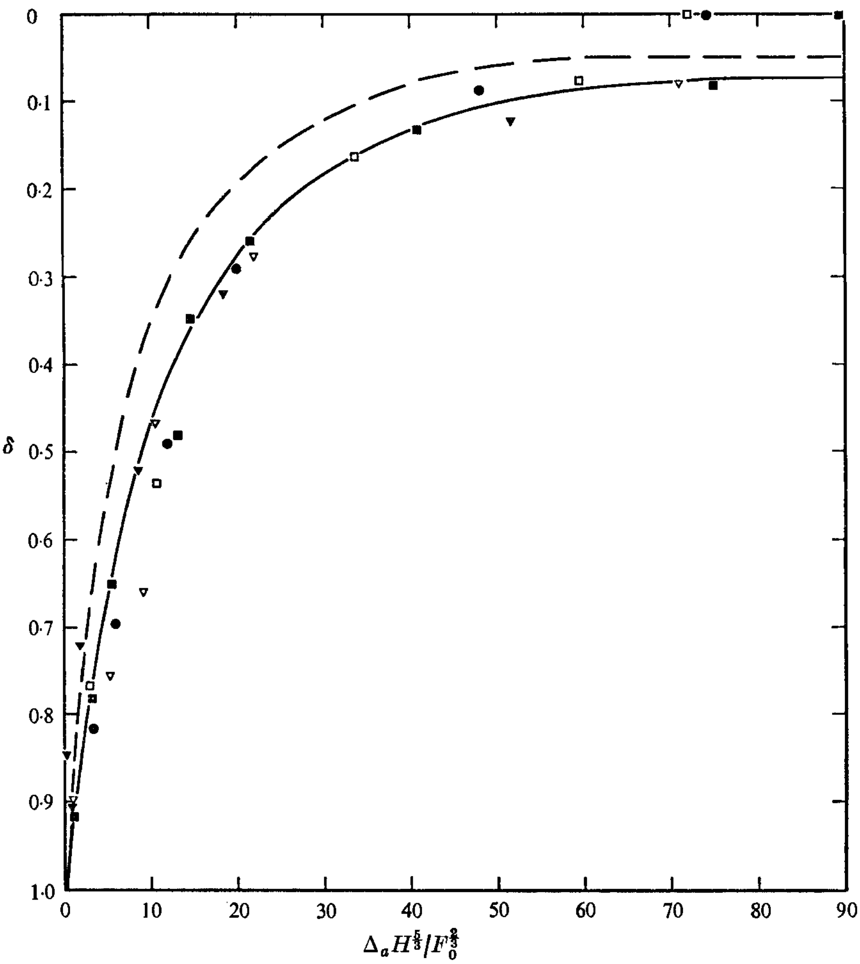

FIGURE 4. Dimensionless asymptotic buoyancy distribution. ————, theory for no flux through interface, $B = 0$; ——, theory for $B = 0{\cdot}3$.

	●	■	▼	□	▽
B	0·328	0·318	0·297	0·270	0·231
Fr	1·378	1·310	0·997	0·869	0·772

the atmosphere as illustrations. Both involve plumes of 6 m characteristic diameter at ground level which pass through a stationary air layer 600 m in depth. Upon reaching the top of the layer the width will be of the order of 70 m and most of the volume flux $\pi w_1 b_1^2$ will have come from entrainment. At the top of the layer an inversion with a temperature decrease of 5 °C is assumed to exist.

In the first example the plume is produced by a patch of ground heated by solar radiation. A buoyancy flux $F_0 = 90\,\mathrm{m}^4/\mathrm{s}^3$ would be typical, leading to a Froude number at the interface of 0·06 and $Q^*/\pi w_1 b_1^2 \ll 0{\cdot}001$. The interface is effectively impermeable to this plume.

Consider a typical plume from a thermal generating station as a second example. A buoyancy flux $F_0 = 4400\,\mathrm{m}^4/\mathrm{s}^3$ produces a Froude number of 2·4.

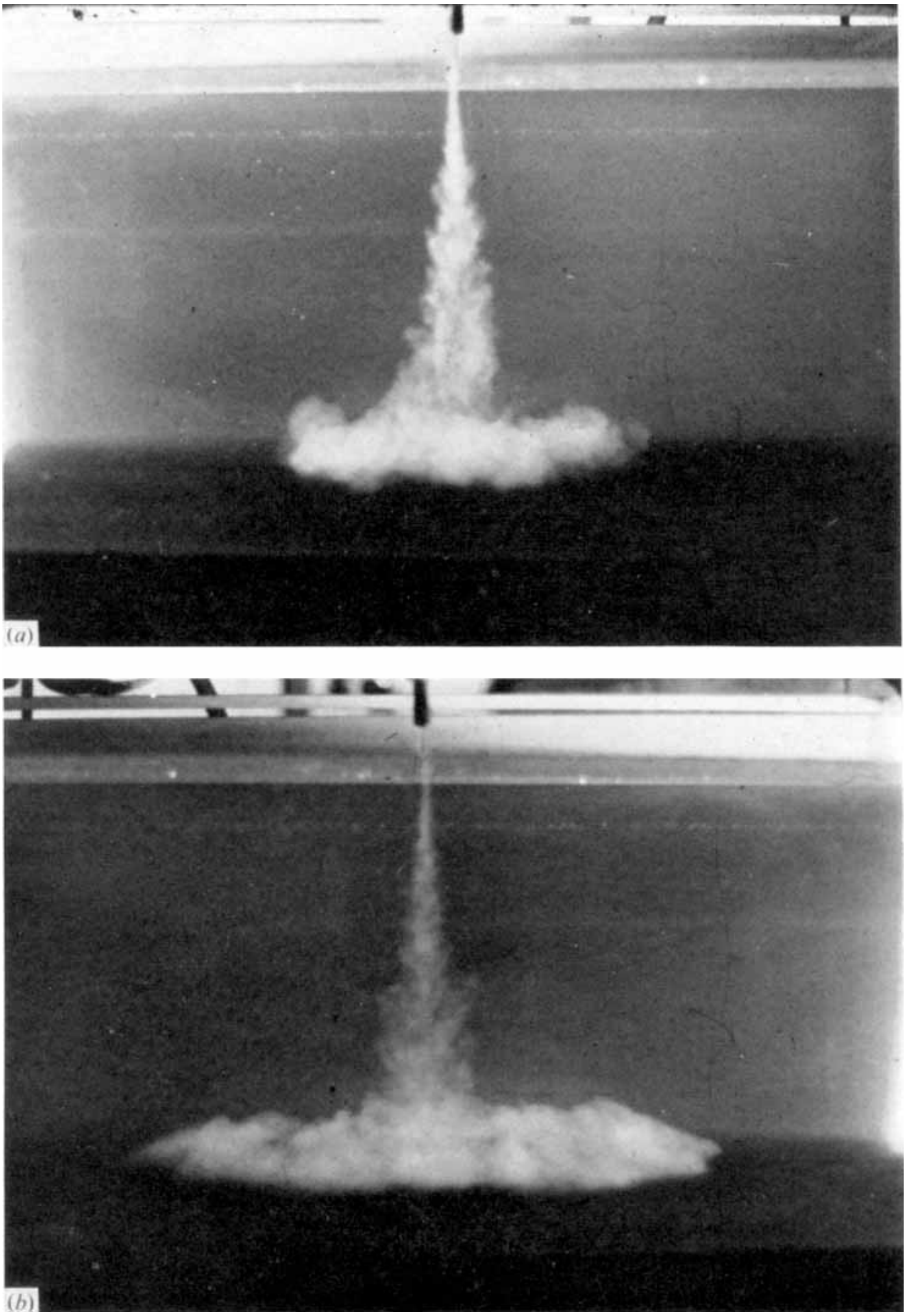

FIGURE 1. Plume of dyed fluid (*a*) immediately after striking interface and (*b*) 2 s later. Plume flux is 1 ml/s of salt solution, specific gravity 1·12. Heavier layer is salt solution, specific gravity 1·084, dyed blue. Froude number = 0·55.

From figure 2 this gives $Q^*/\pi w_1 b^2 \simeq 0{\cdot}3$. The flow of mixed fluid into the environment is thus increased by 30 %, which means that the concentration of pollutant is reduced by 30 %. This would be a significant improvement in practice because these conditions of static warm surface layers are those with the greatest pollution dangers.

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